STABILISATION OF SINGLE LINK MANIPULATOR

A PROJECT REPORT

Submitted by

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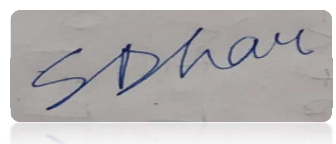
ABSTRACT

The mathematical modeling and stabilisation of a single link manipulator is examined and simulated. The model is based on a set of nonlinear second-order ordinary differential equations and to simulate the dynamics accurately Lagrangian and Euler-Lagrange equations were successfully derived and reproduced.

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my guide, Prof. Aparajita Sengupta her valuable guidance, consistent encouragement, personal caring, timely help and providing me with an excellent atmosphere for doing research. All through the work, in spite of her busy schedule, she has extended cheerful and cordial support to me for completing this project.



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WINTER INTERNSHIP CERTIFICATE

Name SOMIK DHAR

has accomplished an internship from **01.12.2019** to **15.01.2020**(**6 WEEKS**) on "STABILISATION **OF SINGLE LINK MANIPULATOR**" under my supervision as a part of his 4-year B.Tech. undergraduate programme in the Department of Electrical Engineering.

Contents of the internship:

The work examines the model and simulates the closed loop control for stabilisation of a single link manipulator. The model is in the form of a set of nonlinear second order ordinary differential equations. The derivation is done using the Lagrangian and Euler-Lagrange equations. The controller successfully test in a simulation platform.

Comments:
Somik was regular in his work. The internship report submitted by him entitled is original in nature and fulfils the requirements of a winter-internship programme.

Place, Date 16.1.20

Aparajita Sengupta

Signature of the supervisor of the internship provider

stamp

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LIST OF SYMBOLS

q	Independent coordinates
$\mathcal{K}(q,\dot{q})$	Kinetic Energy ,[J]
$\mathcal{P}(q)$	Potential Energy, [J]
$\mathcal{L}(q,\dot{q})$	Lagrangian function, [J]
m	mass, [kg]
g	gravity constant, [Nm^{-2}]
f	force vector, [N]
θ	Joint angle position of the arm, [rad]
$\ddot{\theta}$	Accelerator of the arm, [ms^{-2}]
x, y	Positions of the link, [m]
v	velocity of the link, [ms^{-1}]
τ	Actuator torque, [Nm]
τ_{PID}	The Torque of the Controller output, [Nm]
K_P	The proportional gain for the arm
K_i	The integral gain for arm
K_d	The derivative gain for arm
A	system matrix
B	input matrix
C	output matrix
D	feedforward matrix
U	input or control vector

1 INTRODUCTION

A robot manipulator is an electronically controlled mechanism, consisting of multiple segments, that performs tasks by interacting with its environment. They are also commonly referred to as robotic arms. The use of mobile manipulators is exponentially increasing in different fields due to factors related to human security. Many real-life application environments may be dangerous to human beings or cannot be reached, such as high-temperature sites or ones where harmful gasses are present. The main objective of the manipulator is to reach a certain location and pick up objects. There are two scenarios of using mobile manipulators in industrial fields. The first scenario entails using robot manipulators in transporting and moving objects and tools in known environments. The second one entails using the robots in unstructured environments, especially in dangerous sites that are unsuitable for human beings.

Control of robot manipulators is a very interesting field due to its complex dynamical model. The dynamical analysis of the robotic model investigates a coupling relation between the joint torques applied by the actuators and the angular positions of the robotic arm. The non-linear dynamics and the coupling relations make accurate and robust control difficult. So that, designing a controller by the traditional control methods that depend on the robotic system dynamics is a very difficult task.

2 Background

2.1 Lagrangian Formulation

The first step in the Lagrangian formulation of dynamics[1] is to choose a set of independent coordinates $q \in R^n$ that describes the configuration of the system. The coordinates q are called generalized coordinates. Once generalized coordinates have been chosen, these then define the generalized forces $f \in R^n$. The forces f and the coordinate rates $\dot{q}$ are dual of each other as due to the fact that there inner product $f^T\acute{q}$ corresponds to power. A Lagrangian function $\mathcal{L}$(q,$\acute{q}$) is then defined as the overall kinetic energy of the system $\mathcal{K}$(q,$\acute{q}$) minus the potential energy $\mathcal{P}$(q),

$$\mathcal{L}(q, \dot{q}) = \mathcal{K}(q, \dot{q}) - \mathcal{P}(q) \tag{1}$$

The equation of motion can be now expressed in terms of Lagrangian formulation as follows:

$$f = \frac{d\partial\mathcal{L}}{dt\partial\dot{q}} - \frac{\partial\mathcal{L}}{\partial t} \tag{2}$$

The equations 1,2 are also referred to as the **Euler-Lagrange** equations with external forces.

3 Mathematical model of Single Link Manipulator

The dynamics of a robot arm is explicitly derived based on the Lagrange-Euler formulation to elucidate the problems involved in dynamic modelling. Figure 1 shows a schematic diagram of a single link manipulator, joint displacement is θ, link lengths l and m represent the mass of the link and τ is the torque for the link. In the model the following assumptions are made:

i The actuators dynamics (motor and gear boxes) is not taken into account.

ii The effect of friction forces is assumed to be negligible.

iii The mass of each link is to be concentrated at the end of the link.

The single link manipulator system is as shown in Figure 1.

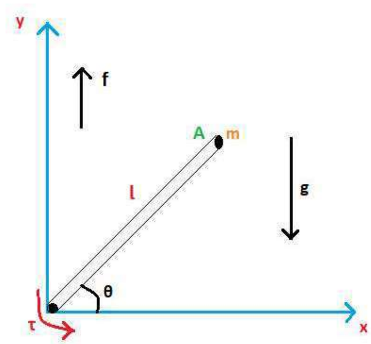

Figure 1: Single link Manipulator

Suppose that the gravitational force mg acts downward, and an external force f is applied upward. By Newton's second law, the equation of motion for the particle is

$$f - mg = m\ddot{x} \tag{3}$$

We now apply the Lagrangian formalism to derive the same result.The kinetic energy is $\frac{1}{2}\mathbf{m}\dot{x}^2$, the potential energy is ***mgx*** and the Lagrangian is:

$$\mathcal{L}(x,\dot{x}) = \mathcal{K}(x,\dot{x}) - \mathcal{P}(x) = \frac{1}{2}m\dot{x}^2 - mgx \tag{4}$$

The equation of motion can be written as by differentiating eq. 4 with respect to time t:

$$f = \frac{d\partial\mathcal{L}}{dt\partial\dot{x}} - \frac{\partial\mathcal{L}}{\partial t} = mg + m\dot{\dot{x}} \tag{5}$$

We now derive the dynamic equations for a single manipulator link moving in the presence of gravity (Figure1). The position and velocity of the link mass are then given by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l\cos\theta \\ l\sin\theta \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l\sin\theta \\ l\cos\theta \end{bmatrix} \dot{\theta} \tag{7}$$

We choose the joint coordinates θ as the generalized coordinates. The generalized forces τ then corresponds to joint torques i.e. f = τ (since $\tau^T\dot{\theta}$ corresponds to power).

The kinetic energy of the system is calculated as

$\mathcal{K}$ = $\frac{1}{2}$mv^2 = $\frac{1}{2}$m$(\dot{x}^2 + \dot{y}^2)$

$$\mathcal{K} = \frac{1}{2}ml^2\dot{\theta}^2 \tag{8}$$

The potential energy of the system is calculated as

$$\mathcal{P} = mgy = mgl\sin\theta \tag{9}$$

The Euler-Lagrange equation for this system as discussed above is given by:

The Lagrangian is given by

$$\mathcal{L} = \mathcal{K} - \mathcal{P} = \frac{1}{2}ml^2\dot{\theta}^2 - mgl\sin\theta \tag{10}$$

By eq.5,

$$\tau = \frac{d\partial\mathcal{L}}{dt\partial\dot{x}} - \frac{\partial\mathcal{L}}{\partial t} = ml^2\ddot{\theta} + mlg\cos\theta \tag{11}$$

4 CONTROL STRATEGY

Proportional-integral-derivative controller (PID) is designed for effective control of the robot arm.The main objective is to make the robot arm move or stop in the desired position. To achieve the stated objective we defined a desired (set point) joint angle θ^d and the objective of robot control is to design the input torque in eq.(14) such that the regulation error is defined as:

$$\theta_e = \theta^d - \theta \tag{12}$$

The PID control law is defined in terms of θ_e as :

$$\tau_{PID} = K_p\theta_e + K_d\dot{\theta}_e + K_i\int_0^t \theta_e dt \tag{13}$$

Here,

θ^d: Desired joint angle[rad]

θ_e: Angle error[rad]

θ: The actual joint angle[rad]

The closed loop system of the single link manipulator is as shown in Fig.2

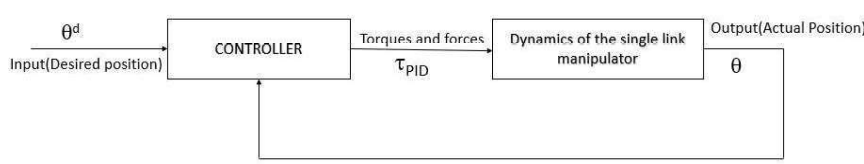

Figure 2: Idealised closed loop block diagram of the manipulator

The closed loop equation of the robot arm is obtained by substituting the control action τ_{PID}

in eq.13 into the robot model(10).

$$ml^2\ddot{\theta} + mlg\cos\theta = K_p\theta_e + K_d\dot{\theta}_e + K_i\int_0^t \theta_e dt = \tau_{PID} \tag{14}$$

4.1 Linearization of the robotic model

As discussed in eq.10, the robotic model is given by:

$$\tau = ml^2\ddot{\theta} + mlg\cos\theta$$

Let the states be defined as follows:

$$x_1(t) = \theta(t)$$
$$x_2(t) = \dot{\theta}(t)$$

Let the input be $u(t) = \tau(t)$ and the output be $y(t) = \theta(t)$.

Therefore the state space model[2] is given by as follows:

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ u(t) - mglcos(x_1(t)) \end{bmatrix} \tag{15}$$

Since the system is non-linearized (due to the presence of $\cos\theta$), so let the system be linearized about $\theta = 45°$(or $\pi/4$ rad.).

Let $X_1{}^* = \theta = \pi/4$ and $X_2{}^* = 0$ be the reference states.

So ***State variables*** are:

$$X_1(t) = x_1(\text{t}) \text{ - } X_1^*$$
$$X_2(t) = x_2(t) \text{ - } X_2^* = x_2(t)$$

Now $\cos x_1(t) = \cos X_1(t) + X_1^* = \cos X_1(t) + \pi/4$.

By Taylor's Series expansion:

$$\begin{aligned} \cos\left(X_1(t) + \pi/4\right) &\approx \cos\pi/4 + (-\sin\pi/4)X_1(t) \\ &\approx \frac{1 - X_1(t)}{\sqrt{2}} \end{aligned} \tag{16}$$

So the linearized ***state space*** model becomes:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{mgl}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} u - \frac{mgl}{\sqrt{2}} \end{bmatrix} \tag{17}$$

Eq.17 can be written as:

$$\dot{X} = AX + BU \tag{18}$$

Also,

$$Y = CX + DU \tag{19}$$

Here, $C = [1 \quad 0]$ and $D = [0]$.

5 Results and Discussion

Simulink model[3] of single manipulator link is prepared based on the Lagrangian and Lagrange-Euler formulation derived in the equation 1 to 8 and the PID controllers are implemented from the equation 11. The parameter values for single link manipulator presented in Table 1 are used for the simulation.In Simulink toolbox PID block is available which is implemented to control the joint angle. The tuning of control parameters is done using PID tuner and the best performance of the controller parameter values are presented in Table 2.

Table 1: Parameters of Single Link Manipulator

Parameters	Link	Unit
m	1	[kg]
l	0.3	[m]
g	9.81	[m^{-2}]

Corresponding to Table1, eq.17, the state space model is given by:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2.081 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} u - 2.081 \end{bmatrix} \tag{20}$$

Here,

$$A = \begin{bmatrix} 0 & 1 \\ 2.081 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = [1 \quad 0] \qquad D = [0]$$

So the transfer function with A,B,C,D parameters as above is

$$T(s) = \frac{1}{s^2 - 2.081} \tag{21}$$

As we can see in eq.21, the system is unstable as one of the poles is in the right hand side of s-plane.

In Simulink toolbox[3] PID block is available which is implemented to control the joint angle. The tuning of control parameters is done using PID tuner and the best performance of the controller parameter values are presented in Table2.

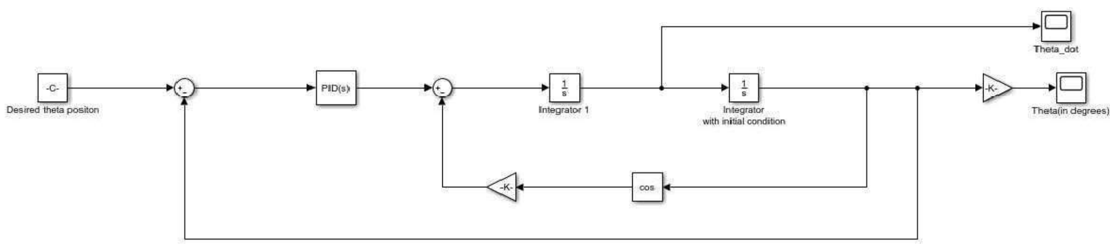

Figure 3: Simulation model for single link manipulator

Table 2: PID controller parameter for single manipulator link

Parameters	Link
K_p	54.5
K_d	48.5
K_i	15.3

In the PID block in simulink, there is an additional term $\mathcal{N}$, which is called filter coefficient is taken as 60 (As practical differentiator is not feasible, a filter coefficient is added in the difference term).

The model is simulated and validate with a different range of joint angles as indicated in Fig.4 to Fig.7.

In fig.4, the initial joint angle is 0°and the desired joint angle is 45°.

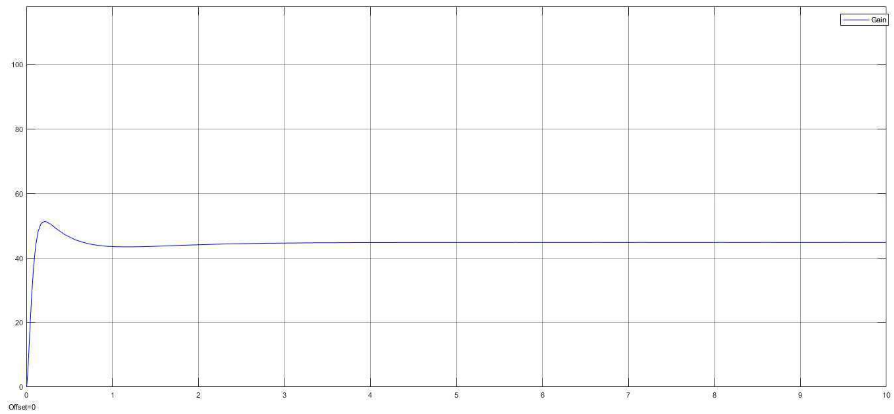

Figure 4: Angle of the single link manipulator over time

In fig.5,the initial joint angle is 0°and the desired joint angle is 90°.

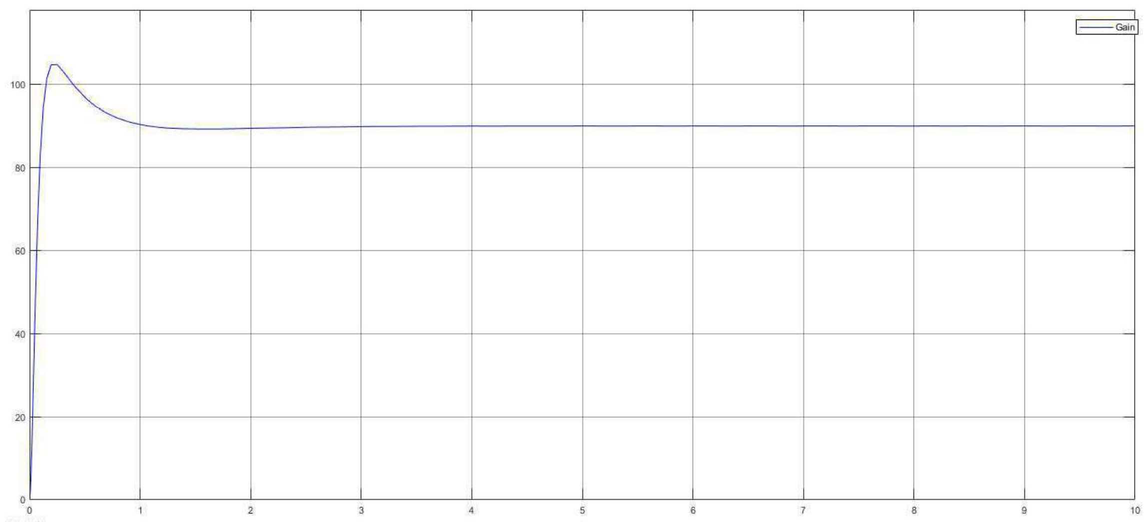

Figure 5: Angle of the single link manipulator over time

In fig.6,the initial joint angle is 60°and the desired joint angle is 45°.

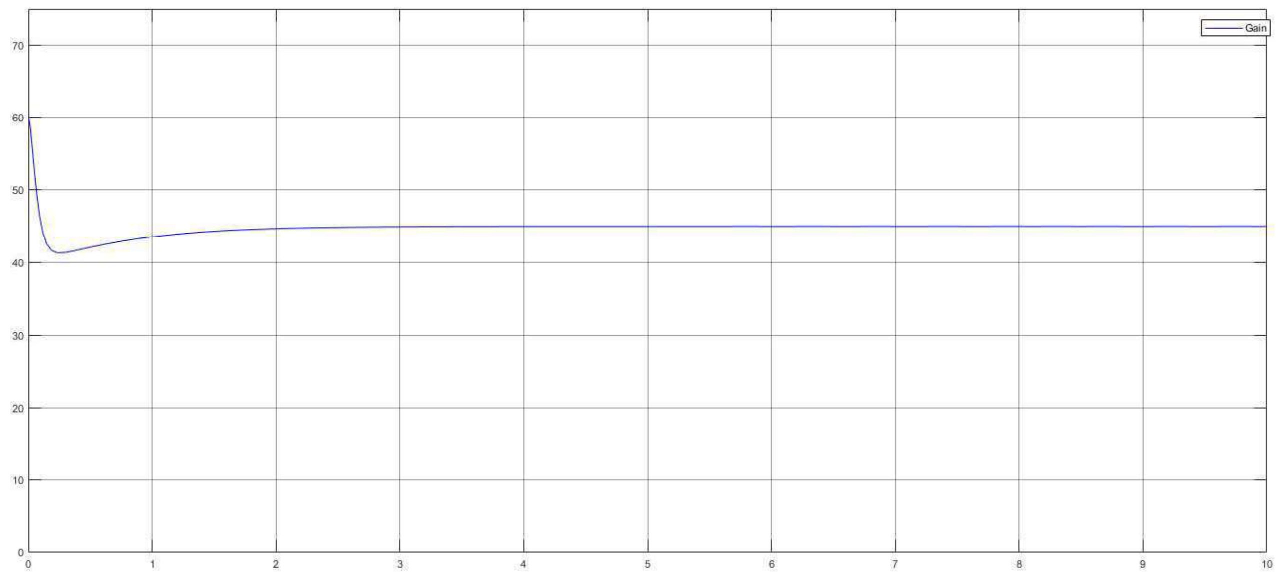

Figure 6: Angle of the single link manipulator over time

In fig.7,the initial joint angle is 90°and the desired joint angle is 30°.

Figure 7: Angle of the single link manipulator over time

6 Conclusion

From the results obtained from fig.4 to fig.7, we observe that the single link manipulator was controlled to reach and stay within a desired joint angle position through implementation and simulation of PID controllers using MATLAB/Simulink. We linearised the dynamic model at an angle of 45°and run the model at angles other than 45°and the results show that our approximation was working at angles other than 45°

7 Future Scope of Work

This work was done on the linearised model of the manipulator. The control strategy may be tested on the actual non-linear model. Also it may be implemented on a lab-scale prototype.

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